



# Object-Part Learning with Local Capsule Hierarchies

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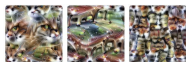
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## Motivation

- Natural scenes have intrinsic structures like repetition of parts and spatial relationships.
- CNNs don't explicitly model this and are thus **uninterpretable** and **non viewpoint-invariant**.
- Capsules [2, 4] seek to model this structure in scenes by composing objects out of progressively more meaningful parts.
- We propose Local Capsule Hierarchies (LCH): an unsupervised generative model based on **hierarchical sparse coding** [3] which is inspired by the visual cortex of the brain.



## Method – Single layer

A single-layer decomposes an image into a **sparse linear combination**  $\sigma$  of  $N$  **parts** with non-negative amplitudes  $\sigma$ .

Each part has its own **deformation parameter** represented by a unit vector  $\theta$ . We model the deformation by decomposing the part into a linear combination in  $\theta$  of  $G$  **learned basis functions**  $\Phi$ .

$$I = \sum_{i=1}^N \sigma_i \sum_{j=1}^G \theta_{ij} \Phi_{ij} + \varepsilon$$

The layer outputs  $\sigma$  and  $\theta$  are determined by minimizing a cost function comprised of reconstruction error (I) and a sparsity penalty for  $\sigma$  (II), while enforcing  $\sigma$  to be non-negative and letting  $\theta$  only vary in its angle.

$$(\hat{\sigma}, \hat{\theta}) = \arg \min_{\sigma \geq 0, \|\theta\|_2=1} \left\| I - \underbrace{\sum_{i=1}^N \sigma_i \sum_{j=1}^G \theta_{ij} \Phi_{ij}}_{(I)} \right\|_2 + \lambda \underbrace{\sum_{i=1}^N \sigma_i}_{(II)}$$

## Method – Optimization

In practice, simply optimizing this cost function using **gradient descent** or ISTA/FISTA is **unstable**.

A solution is to use the **Subspace Locally Competitive Algorithm** [1] which is inspired by neuroscience. We first define the latent variable  $u$ :

$$u_{ij} = \sigma_i \theta_{ij} + \lambda \theta_{ij}$$

Then we optimize  $u$  using the following **dynamical system** which introduces **competition** between the parts through the inhibition term (III):

$$\tau \frac{du_{ij}}{dt} = \langle I, \Phi_{ij} \rangle - \underbrace{\sum_{nm \neq ij} \sigma_n \theta_{nm} \langle \Phi_{ij}, \Phi_{nm} \rangle}_{(III)} - u_{ij}$$

The **basis functions** are learned through gradient descent on the reconstruction error while the inner optimization is unrolled with a small fixed number of iterations.

## Results

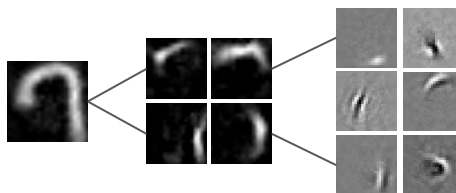


Figure 1: An example of LCH decomposition on the digit “7” from the MNIST dataset. The digit decomposes into mid-level parts which decompose into low-level parts

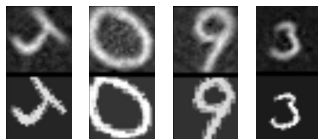


Figure 2: The reconstruction (top) versus the target (bottom) for the LCH training.

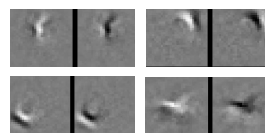
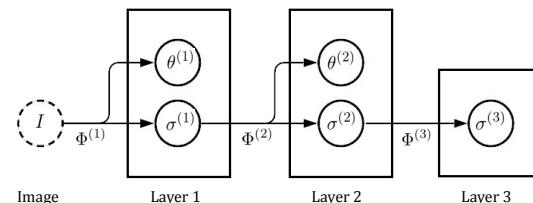


Figure 3: 4 LCH group examples with group size 2

## Method – Multilayer

The higher layers decompose only the amplitudes  $\sigma$ , which can be interpreted as a local pooling operation over the learned deformation  $\theta$ .



## Future Work

- We seek to test this algorithm with downstream tasks such as classification or object detection. Given parse trees of scenes, LCH may prove to have certain strengths.
- Extending this to other datasets will test the robustness of LCH. Furthermore, adversarial analysis of LCH will expose potential strengths.
- Testing data efficiency of this method and experimenting with higher-level basis steering can show new applications.

## Citations

- [1]: Dylan M. Paiton, Steven Shepard, Kwan Ho Ryan Chan, and Bruno A. Olshausen. 2020. Subspace Locally Competitive Algorithms. In *Proceedings of the Neuro-inspired Computational Elements Workshop (NICE '20)*. Association for Computing Machinery, New York, NY, USA, Article 9, 1–8.
- [2]: Sara Sabour, Nicholas Frosst, and Geoffrey E. Hinton. 2017. Dynamic routing between capsules. In *Proceedings of the 31st International Conference on Neural Information Processing Systems (NIPS'17)*. Curran Associates Inc., Red Hook, NY, USA, 3859–3869.
- [3]: Olshausen BA, Field DJ. 1997. Sparse Coding with an Overcomplete Basis Set: A Strategy Employed by V1? *Vision Research*, 37: 3311-3325.
- [4]: Adam R. Kosiorek, Sara Sabour, Yee Whye Teh, and Geoffrey E. Hinton. 2019. Stacked capsule autoencoders. *Proceedings of the 33rd International Conference on Neural Information Processing Systems*. Curran Associates Inc., Red Hook, NY, USA, Article 1390, 15512–15522.